point further we rewrite equation (4) in dimensionless form as

$$du^+ = \frac{dy^+}{l^+}, \tag{6}$$

and with $l^+ = \kappa(y^+ + (\delta y_0)^+)$ integrate with $u^+ = 0$ at $y^+ = 0$ to give

$$u^+ = \frac{1}{\kappa}\ln\left(\frac{y^+}{(\delta y_0)^+}+1\right). \tag{7}$$

Use of $(\delta y_0)^+ = 0.0307 Re_k$, and $\kappa = 0.41$ gives

$$u^+ = \frac{1}{\kappa}\ln\left(\frac{y}{k_s}\right)+8.5, \tag{8}$$

and the constant 8.5 is seen to be identical to Nikuradse's value (though Nikuradse did recommend $\kappa = 0.40$). Equation (8) is also shown in Table 1, and agrees with the velocity profile obtained from equation (5) with $(\delta y_0)^+ = 0.0307(Re_k - 46)$ to within 2%. Thus we conclude that if equation (3) is used to calculate the velocity profile, $(\delta y_0)^+ = 0.0307(Re_k - 46)$ is appropriate, while if equation (4) is used $(\delta y_0)^+ = 0.0307 Re_k$ is appropriate; also equation (1) does not contain any significant information beyond that which is contained in Nikuradse's fully rough velocity profile. The effect of the constant -46 in equation (1) is to approximately cancel the effect of ν in equations (2) and (3) so as to have a negligible effect of viscosity on the fully rough velocity profile. Thus we cannot agree with the assertion following equation (3) of ref. [1] that the implicit viscosity dependence in equation (1) results from viscosity not having a completely negligible effect on fully rough hydrodynamic behavior: the alleged viscosity dependence has been shown to be present simply because it is required to cancel the viscosity term in the boundary layer equations, *if that viscosity term is retained.* If the viscosity is not retained in the boundary layer equations for a fully rough wall, the constant -46 in equation (1) is not required, and the required mixing length offset can be deduced directly from Nikuradse's velocity profile, or its equivalent.

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REFERENCES

1. P. M. Ligrani, W. M. Kays and R. J. Moffat, A heat transfer prediction method for turbulent boundary layers developing over rough surfaces with transpiration, *Int. J. Heat Mass Transfer* **24,** 774–778 (1981).
2. P. M. Ligrani, R. J. Moffat and W. M. Kays, The thermal and hydrodynamic behavior of thick, rough-wall turbulent boundary layers, Report No. HMT-29, Thermosci. Div., Dept. of Mechanical Engineering, Stanford University (1979).
3. M. E. Crawford and W. M. Kays, STAN 5—A program for numerical computation of two dimensional internal/external boundary layer flows, Report No. HMT-23, Thermosci. Div., Dept. of Mechanical Engineering, Stanford University (1975).
4. M. M. Pimenta, R. J. Moffat and W. M. Kays, The turbulent boundary layer: an experimental study of the transport of momentum and heat with the effect of roughness, Report No. HMT-21, Thermosci. Div., Dept. of Mechanical Engineering, Stanford University (1975).

REPLY TO "COMMENTS ON 'A HEAT TRANSFER PREDICTION METHOD FOR TURBULENT BOUNDARY LAYERS DEVELOPING OVER ROUGH SURFACES WITH TRANSPIRATION'"

MOST of the correct remarks made by Mills and Hang regarding the closure model of ref. [1] have been taken from our private communications with them. A reply is required in order to clarify their incorrect statements, and to avoid any confusion about the mixing-length closure scheme given in ref. [1]. It should also be mentioned that the use of equations (6)–(8) in the comment is a rearranged form of derivations presented in refs. [2, 3].

Referring to the last sentence of the comment, it is not logical to discard viscosity (viscous stress) terms if they are actually *non-negligible.* What is needed, and what is provided in ref. [1], is a viscosity-dependent mixing length which will reproduce the observed Reynolds number independence of the equation

$$U^+ = \frac{1}{\kappa}\ln(y/k_s)+8.5. \tag{1}$$

As shown by results in refs. [2, 4], equation (1) is valid in log-regions of boundary layers developing over impermeable uniform spheres roughness when $Re_k > 55$. Reference [2] also shows that equation (1) is faithfully reproduced by the closure model described in ref. [1].

The viscosity independence of equation (1) results from near cancellation of the direct effects of viscosity on turbulent diffusion for an impermeable surface. However, Mills and Hang have confused the fact that this near cancellation is required because viscosity is present and non-negligible, even for Re_k values well above the fully rough threshold value of 55. The fact that viscosity is not insignificant is evident from results in Fig. 1, where ν/ε_M is shown as determined from measurements presented in refs. [2, 4]. These results are in excellent agreement with ν/ε_M from the mixing-length equations of ref. [1].

Towards the end of their comment, Mills and Hang have disagreed over a statement which we never made. In doing so, these individuals have missed two important points made clear after equation (3) in ref. [1]. The first is that the fully rough mixing length retains a viscosity dependence, which arises from the $Re_k' = 46$ term in equation (2) of ref. [1]. Secondly, the viscosity dependence of fully rough hydrodynamic properties, such as the mixing length and eddy diffusivity for momentum, ε_M, is expected to be most evident for roughness Reynolds numbers ranging from $Re_{k,R}$ to about 200.

We agree that, on *impermeable* rough walls, our viscosity-dependent mixing length almost exactly cancels the effect of the viscous shear stress, thus reproducing the observed Reynolds number independence of equation (1) for roughness Reynolds numbers above about 55. Clearly, the same result is obtained by ignoring both the mixing-length viscosity-dependent terms and the effect of viscosity on the diffusion of momentum. However, the cancellation is unlikely to persist in the case of transpiration for which our model was intended,

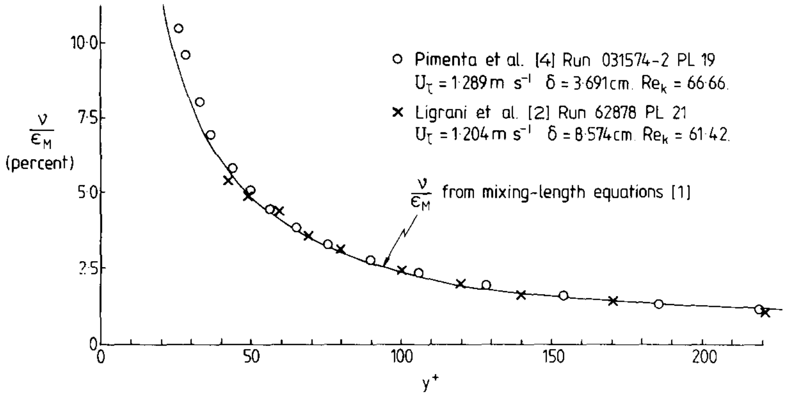

FIG. 1. Variation of ν/ε_M in near-wall regions of fully rough turbulent boundary layers.

and we would expect our physically realistic model to perform much better than if all viscous effects are unscientifically omitted. Viscosity independence of bluff body flows does not normally occur at roughness Reynolds numbers as low as 55, so it is clear that the flow over an impermeable wall having uniform spheres roughness requires more care in turbulence modelling than Mills and Hang seem to think.

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REFERENCES

1. P. M. Ligrani, W. M. Kays and R. J. Moffat, A heat transfer prediction method for turbulent boundary layers developing over rough surfaces with transpiration, *Int. J. Heat Mass Transfer* **24**, 774–778 (1981).
2. P. M. Ligrani, R. J. Moffat and W. M. Kays, The thermal and hydrodynamic behaviour of thick, rough-wall turbulent boundary layers, Report No. HMT-29, Thermosci. Div., Dept. of Mechanical Engineering, Stanford University (1979).
3. W. M. Kays and M. E. Crawford, *Convective Heat and Mass Transfer* (2nd edn.). McGraw-Hill, New York (1980).
4. M. M. Pimenta, R. J. Moffat and W. M. Kays, The turbulent boundary layer: an experimental study of the transport of momentum and heat with the effect of roughness, Report No. HMT-21, Thermosci. Div., Dept. of Mechanical Engineering, Stanford University (1976).

COMMENTS ON 'SOME PROPERTIES OF THE COEFFICIENT MATRIX OF THE DIFFERENTIAL EQUATIONS FOR PARALLEL-FLOW MULTICHANNEL HEAT EXCHANGERS'

I SHOULD like to make some comments on the information given in the short communication by L. Malinowski, *Int. J. Heat Mass Transfer* **26**, 316 (1983).

Malinowski states that the general solution of the matrix differential equation for parallel-flow heat exchangers

$$\frac{dt}{dx} = At, \tag{1}$$

as given by Wolf [1], is not a general solution at all, even with the complement given in ref. [2], since multiple non-zero latent roots of A may occur.

This is not entirely true, but the main point is that the general solution is already known.

In the mid 1970s it was proved that for $\Sigma W_i \neq 0$, matrix A has one, and for $\Sigma W_i = 0$ two, and only two, zero latent roots [2, 3]. More recently Zaleski proved that all the blocks of the Jordan canonical matrix J, corresponding to the non-zero latent roots of A, are of the first order, irrespective of a possible multiplicity of the latent roots [4].

The general solution to equation (1) is

$$t = K\,e^{Jx}C, \tag{2}$$

where K is the matrix transforming A to the Jordan canonical form $A = KJK^{-1}$, and C is a vector of constants. Hence it is clear that for $\Sigma W_i \neq 0$, J is a diagonal matrix while for $\Sigma W_i = 0$ it has one second-order block, corresponding to zero latent roots, and all the others are of the first order.

The general solution (2) provides the basis for computations of the thermal performance of parallel-flow heat exchangers. However, such computations are practically possible only if the number of channels is notably restricted, as otherwise they can be prohibitively time-consuming and difficult to carry out, particularly if matrix A happens to be ill conditioned. This last situation occurs when channels must be counted in tens or more, as is frequently found with plate heat exchangers. In such cases the approximate method developed by Settari and Venart [5] for solving equation (1) proves more effective [6].

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